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63/1 (SEM–6) DSE4/MATHE6046

2024

MATHEMATICS

Paper : MATHE6046

(Mechanics)

Full Marks : 80

Pass Marks : 32

Time : Three hours

The figures in the margin indicate full marks for the questions.

1. Choose the correct answer : ***(any six)*** $1\times6=6$

   (i) If a couple is in equilibrium, what can be said about the magnitudes of the two forces constituting the couple ?

   (A) They can be equal or unequal

   (B) They must be unequal

   (C) They must be equal

   (D) Their magnitudes are irrelevant for equilibrium

Contd.

(ii) What is the relationship between the direction of the resultant and the direction of the forces in a system?

(A) The resultant always points in the same direction as one of the forces

(B) The resultant is independent of the directions of the individual forces

(C) The resultant points in the direction opposite to the largest force

(D) The resultant is determined by the average direction of the forces

(iii) According to Coulomb's law of friction, the frictional force between two surfaces is directly proportional to

(A) The coefficient of friction

(B) The angle of inclination

(C) The speed of the object

(D) The mass of the object

(iv) Which force is always included in a free body diagram, regardless of the situation?

(A) Frictional force

(B) Tension force

(C) Gravitational force

(D) Magnetic force

(v) What is the relationship between the distance traveled by the centroid and the area of a shape in the theorem of Pappus-Guldinus?

(A) They are inversely proportional

(B) They are directly proportional

(C) They are unrelated

(D) They are equal

(vi) How does the presence of non-conservative forces, such as friction, affect the conservation of mechanical energy in a system?

(A) It has no effect on the conservation of mechanical energy

(B) It converts mechanical energy into other forms of energy

(C) It increases the mechanical energy of the system

(D) It decreases the mechanical energy of the system

(vii) What is translation in the context of rigid body motion?

(A) Movement of an object along a curved path

(B) Movement of an object while rotating around an axis

(C) Movement of an object without changing its orientation

(D) Movement of an object with changing its shape

(viii) How does the centre of mass of a rigid body behave during translation and rotation?

(A) It moves along a straight path during translation and remains fixed during rotation

(B) It remains stationary during translation and rotates during rotation

(C) It moves along a curved path during translation and rotates around an axis during rotation

(D) Its behaviour depends on the shape of the rigid body

(ix) What is the first moment of an area with respect to a reference axis?

(A) It measures the area of the shape

(B) It measures the centroid of the shape

(C) It measures the distribution of mass within the shape

(D) It measures the tendency of the area to rotate about the axis

(x) When is the moment of a force about a point considered positive?

(A) When the force causes clockwise rotation

(B) When the force causes counter-clockwise rotation

(C) When the force is applied at the point

(D) When the force is perpendicular to the line connecting the point and the force

2. Answer ***any five*** of the following questions : 2×5=10

(i) State the conditions under which a force will not produce a moment about a point.

(ii) What is the significance of the distance between the forces in a couple ?

(iii) What information does a free body diagram provide about the motion of an object ?

(iv) What is the general equation of equilibrium in mechanics ?

(v) What factors can influence the coefficient of friction between *two* surfaces ?

(vi) Define translation and rotation in the context of rigid body motion.

(vii) Define kinetic energy and its role in the work-energy equation.

3. Answer ***any six*** of the following questions : 5×6=30

(a) If three forces $\vec{P}, \vec{Q}, \vec{R}$ acting at the angular points of a triangle ABC along the tangents to the circum-circle. the same way round, are equivalent to a couple, then show that

$$\vec{P} : \vec{Q} : \vec{R} = \sin 2A : \sin 2B : \sin 2C$$

(b) Prove that if three forces acting upon a rigid body is represented in magnitude, direction, sense and line of action by the sides of a triangle, taken in order, then they are equivalent to a couple whose moment is equal to twice the area of the triangle.

(c) A heavy uniform rod of length $2a$ rests in equilibrium, having one end against a smooth vertical wall, and being placed upon a peg at a distance b from the wall. Show that the inclination of the rod to the horizontal is

$$\cos^{-1}\left(\frac{b}{a}\right)^{\frac{1}{3}}$$

(d) A uniform ladder is in equilibrium with one end resting on the ground and the other against a vertical wall; if the ground and wall be both rough, the coefficients of friction being μ and μ' respectively, and if the ladder be on the point of slipping at both ends, then show that the inclination of the ladder to the horizon is given by

$$\tan\theta = \frac{1-\mu\mu'}{2\mu}$$

(e) Find the relation between the effort and the resistance in the case of a screw, when friction is taken into account.

(f) Given a plane region bounded by the curves $y = x^2$ and $y = 4$, find the volume of revolution when this region rotates about the x-axis using the Pappus-Guldinus theorem.

(g) A roller coaster car with a mass of 500 kg starts from rest at a height of 30 meters above the ground. If the track is frictionless, calculate the velocity of the car at the bottom of the hill.

(h) Prove that the displacement of a rigid body can be achieved by a combination of a translation and a rotation about a fixed axis passing through any point in the body.

(i) In a one-dimensional motion with a conservative force field described by potential $v(x) = 2x^2$, a particle with mass m travels from position x_1 to x_2 at times t_1 and t_2 respectively. The total mechanical energy of the particle is E. Derive an expression for the time difference $t_2 - t_1$ in terms of the mass (m), the potential function, the total energy.

(j) A truck has a speed of 20 km/hr and an acceleration of 3 km/sec^2 at time t. A cylinder of radius equal to 2 ft is rolling without slipping at time t such that relative to the truck it has an angular speed ω and angular acceleration ω of 2 rad/sec and 1 rad/sec^2. Determine the velocity and acceleration of the centre of the cylinder relative to the ground.

4. Answer ***any two*** of the following questions : 10×2=20

(a) The least force which will move a weight up an inclined plane is P. Show that the magnitude of the least force, acting parallel to the plane, which will move the weight upwards is

$$P\sqrt{1+\mu^2}$$

μ being the coefficient of friction of the plane.

(b) A plane surface is bounded by the x-axis, the curve $y^2 = 25x$, and a line parallel to the y-axis. What are the first moments of the area about the x and y axes and what are the centroidal coordinates ?

(c) Establish the general relationship between time derivatives of a vector for different references.

(d) Prove that any system of coplanar forces acting on a rigid body is equivalant to a single force acting at an arbitrarily chosen point together with a single couple.

5. Answer ***any one*** of the following questions : 14

(a) (i) If two couples, whose moments are equal and opposite, act in the same plane upon a rigid body, then prove that they balance one another. 7

(ii) Two couples with forces along the sides of a parallelogram are in equilibrium. Find the ratio of the magnitudes of the forces of the couples. 7

(b) Establish the general equations of equilibrium. A weight W is attached to an endless string of length l which hangs over two smooth pegs, distant c apart in a horizontal line. Prove that the pressure on each peg is

$$W\sqrt{\frac{l-c}{2(l-2c)}}$$

7+7=14

(c) (i) A heavy solid right circular cylinder cone is placed with its base on a rough inclined plane, the inclination of which is gradually increased. Determine whether the initial motion of the cone will be one of sliding or tumbling over. 7

(ii) If the moments of two given intersecting forces about a point in their plane be equal and of the same sense, then prove that the point must be on a certain straight line. 7

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